TIME VALUE OF MONEY

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APTA’s dedicated financial modeling team is led by Santosh Singh. Santosh has more than 12 years of industry experience. With a technical background in drilling engineering and further qualification in Finance and Economics, he has worked in a number of major technical and commercial functions and gained extensive experience in economics evaluation, business development and commercial agreements.

Santosh’s commercial valuation and analysis experience covers Africa, Asia, and Eurasia to name a few. He has a proven ability in the fiscal regime modelling, investment analysis, and providing high quality support to management for the strategic investment decisions.

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TIME VALUE OF MONEY

“Time Value Of Money (TVM): The idea that money available at the present time is worth more than the same amount in the future due to its potential earning capacity. This core principle of finance holds that, provided money can earn interest, any amount of money is worth more the sooner it is received.”

DISCOUNTING

Recall your school days when you were taught the concept of interest rate and compounding. Interest is a charge for borrowing money, stated as a percentage of the amount borrowed for a given period of time. When we deposit money in our Bank, the Bank pays us an interest on the amount deposited in our account. Thus our money keeps 'growing' if we leave it with the bank. Hardly would we have conceived the profoundness of this concept at that time. Interest rate and compounding is at the epicenter of the financial universe. It drives the financial markets and the entire global economy is dependent on it.

Interest can be calculated in two ways, as *simple interest* or as *compound interest*. Simple interest is computed only on the original amount borrowed. It is the return on that principal for one time period. Compound interest is calculated each period on the original amount borrowed plus all unpaid interest to date. Compound interest is interest on the original principal and also *interest on the interest*.

Let's illustrate a case of simple interest first. Let's say you deposit a principal of $1,000 in a bank at the beginning of year 1 and intend to keep it in the bank for 5 years. The bank pays you a simple interest of 5% per annum. At the end of 5th year you will have $1,250 in the bank. Table below shows the summary of you $1,000 deposit (investment) for each year.

Simple Interest	1	2	3	4	5
Amount Deposited	1,000				
Interest Earned	50	50	50	50	50
Balance In Bank A/C	1,050	1,100	1,150	1,200	1,250

Your deposit grows from $1,000 to $1,250 in 5 years, implies interest earning of $250. Using the formula for simple interest, $1,000 becomes $1,100 (= $1000 + $1,000 x 5% x

2) at the end of 2nd year.

Simple Interest

$$I = P \times r \times N$$

P = principal
r = annual interest rate
I = total Interest accrued over N periods
N = total number of years or periods

Using the formula for simple interest, $1,000 becomes $1,100 (= $1000 + $1,000 x 5% x 2) at the end of 2nd year.

Let's move on to a compound interest now. The application of compound interest or interest on interest means growth in an investment from one period to another is due to the interest earned on the original principal *and also* due to the interest earned on the previous period's interest earnings.

Let's assume the bank where you deposited $1,000 instead of paying 5% annual interest, actually pays you 5% annual compound interest. At the end of 5th year you will have $1,276 in the bank. Notice the difference in interest earned each period in this case in the table below.

Compound Interest	1	2	3	4	5
Amount Deposited	1,000				
Interest Earned	50	53	55	58	61
Balance In Bank A/C	1,050	1,103	1,158	1,216	1,276

Your deposit grows from $1,000 to $1,276 in 5 years, implies interest earning of $276.

The $1,000 initial deposit grows to $1,050 ($1000 + 5% x $1000) at the end of year 1. And one year thereafter, or two years from the start it grows to $1,103 which can be explained as below:

$1,103 = $1,000 principle
+ $50 first year interest
+ $50 x 5% interest on first year interest
+ $1000 x 5 interest on principle for 2nd year.

Annual compounding:

$$A = P \times (1 + r)^n$$

P = principle at the start
r = annual interest rate
n = number of years,
n = 1 for 1 year, 2 for 2 years etc.
A = total amount at end of n^{th} year

You can also work out the values in the table above using the compound interest formula in the Blue box above (P = $1,000 r =5% n = 1, 2, …5 years).The example above was to illustrate how simple interest and compound interest works.

In the real world it's the compound interest which rules the world of financial management and investment decisions. This is because banks and other financial institution always lend on the basis of compound interest. Now that we have refreshed our concept of simple and compound interest, we are ready to explore the meaning of 'Time Vale of Money'.

Recall the compound interest rate calculation table above at 5% compound interest rate for 5 years. Assume now I offer you to pay $1,000 today or $1,300 after 5 years from now. I am sure you will gladly accept the option of receiving $1,300 5 years' from now. This is because if you choose to receive $1,000 today, it can grow to $1,276 in 5 years' time, which is less than the second option I offered you. This implies that the value of receiving $1,300 at the end of 5 year is *more* than the value of value of receiving $1,000 today.

What if I had offered the option to receive $1,276 at the end of 5 year from now, vs. receiving $1,000 today? As long as all our previous assumption about the bank's interest rate remains unchanged, you should be indifferent toward both the option. If you choose to accept $1,000 today, you can deposit it in the bank and earn compound interest of 5% for 5 years and by the 5^{th} year end will have $1,276. On the other hand if you opted to accept $1,276 after 5 year, you will have no more than in the first option; i.e. same amount of money in the end. This implies that the value of receiving $1,276 at the end of 5th year is *same* as the value of receiving $1,000 today.

Now let's consider one more scenario (promise this the last one!). What if I offered you the option to receive $1,200 at the end of 5th year from now, vs. receiving $1,000 today? Assumptions about the bank's interest rate remain same. You should select the option to receive $1,000 today because in that case you will have $1,276 by 5th year end by simply keeping it in the bank, more than $1,200 of the alternate option. This implies that the value of receiving $1,200 at the end of 5th year is *less* than the value of receiving $1,000 today.

PV $X = present value of $X as of today

PV $1,300 > $1,000

PV $1,276 = $1,000

PV $1,200 < $1,000

Please note that the above equations stand true under a given interest rate (=r = 5% annual) assumption. If the interest rate assumption changes, the above equation will no longer be valid.

The value of $X today is more than the value of $X in some distant future, for the simple reason that $X of today will earn you some extra $ by keeping it in bank which will pay you an interest on it, or by investing this $X elsewhere which pays a greater return than the interest you would earn by keeping it in the bank. This is the principle behind the so called 'time value of money' and the foundation of any investment analysis.

So what is PRESENT VALUE? Basically present value is the value as of today (or on a given reference point in time). Let $X be an amount that you will receive in future, n years from now. So to you, $X represents the future value. Present worth of that future value (= $X, at end of nth year) today is called PRESENT VALUE. This present value of $X will be equal to an amount that you will need to invest *today* (at a given interest rate) such that your investment of today becomes $X by end of 'nth' year.

*In the example above that we discussed we can safely say that **present value** of $ 1,276 today is $1,000. Alternatively we can say **future value** of todays $1,000 is $1,276 in the 5th year. The key underlying assumption here is that applicable interest rate is 5% (compounded annually).*

If all this is still confusing then please go back to the previous page and re-read it slowly and carefully.

Time value problems generally require us to find out the future value of an investment due to compounding effect. This means we need to project the cash flows forward into the future periods based on an appropriate interest rate. Thus projecting a cash flow into the future simply means we are compounding that particular cash flow based using an appropriate interest rate.

Finding the present value is similar but just in the opposite direction. Instead of projecting the cash flow into future, we have to bring a future period cash flow to today or to any reference date. In other words, it brings back the cash flow to the beginning of the investment life. And this process is termed as 'Discounting'.

Discounting and compounding are reverse of each other; both in meaning and the mathematical formula (which we will see in a moment) .Compounding projects the present cash into the future period. Discounting brings back the future cash into the present period. Compounding involves multiplication with interest rate. Discounting involves division by interest rate.

But before we move on to the formulas for discounting and compounding, let us introduce the concept of 'timeline'. A 'time line' is simply a visual representation of the cash flows associated with different point in time. We explain it below.

A cash flow can be negative or positive. A payment or cash-outflow from the system (or the business concerned) is shown as negative cash flow. Cash outflow leads to reduction in the cash balance of the business concerned. A cash-inflow into the system (or the business) is shown as positive cash flow.

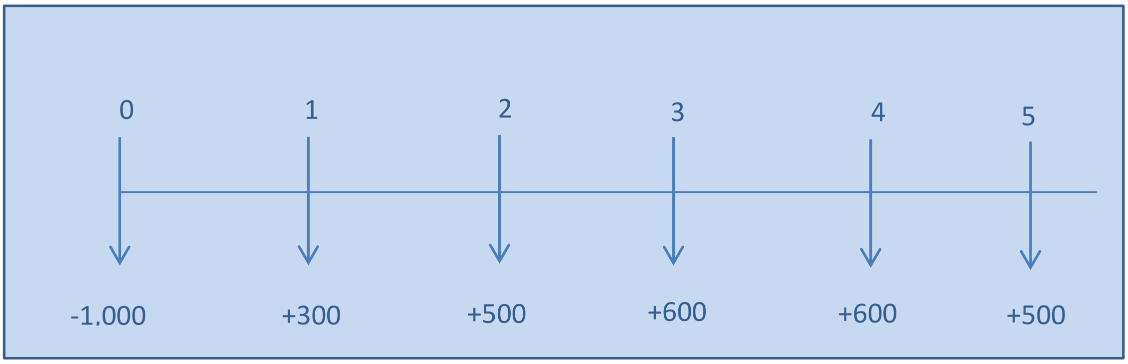
To visualize the situation we draw a straight line, divided into equal intervals. Each interval represents a period. The reference date which in most case is same a present (today) is marked as ZERO (t =0) on the time line. A cash flow that occurs today will be put at time ZERO on the time line. A cash flow that will occur in year 3 will be shown at the mark, t=3, on the time line and so on.

Let's clarify this with an example. Assume that you have started a car rental business by buying a car for $1,000 to rent it to customers. Your car rental business generates cash flow over 5 years as shown in table below. At the end of 5th year you sell your car for $100 and close the business. For simplicity we assume all figures are after-tax.

Year	0	1	2	3	4	5
Bought Car	1,000					

Rental Income		300	500	600	600	400
Car Sold						100
Cash flow ($)	**-1,000**	**300**	**500**	**600**	**600**	**500**

We show the above cash flows on a time line as below:



Once we have placed the cash flows on the time line we can easily 'move' them back and forth depending on what we want to do with them. 'Moving' the cash flows does not mean we will keep shifting their original position on the time line. What we mean is that we can convert them into present value by discounting the future cash flow. Or we can convert the cash flows of the respective period into a final year future cash flow figure which will be the future value of all the cash flow over the life of the project/investment.

One very important point to mention here is that you have make an assumption as whether these cash flow shown on the timeline are at the beginning of the period or at the end of the period. This is very important in recognizing the time value of those cash flows.

Our assumption in this section is that all cash flows occur at the end of the period. So for example, in the timeline above a cash-inflow of $300, occurs at the end of year1 and another cash-inflow of $500 occurs at the end of year 2. Please also note that the end of one period is also the start of next period. So for example we can say a cash-inflow of $600 occurs at the end of year 3 or at the start of year 4. The marking on the time represents end of the year.

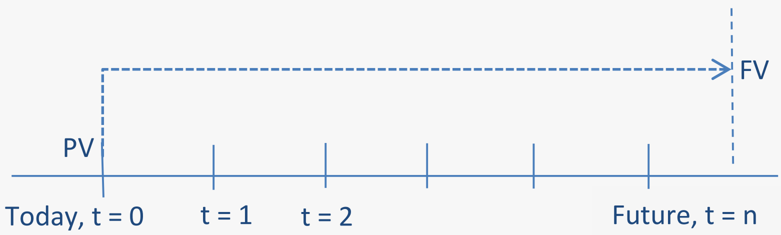
Interest rate is the measure of time value. The higher the interest rate the higher the future value. Looked other way, higher the interest rate, lower the present value.

FUTURE VALUE OF SINGLE SUM

Future value (FV) is the amount to which a current deposit will grow over time when it's deposited in a compound interest paying account.

$$FV = PV \times (1 + r)^n$$

P V= amount invested today
r = rate of return (interest rate) per compounding period
n = total number of compounding period



In the above formula, investment involves a single cash-outflow (PV) today at time zero (t=0). The sum FV is the value of the investment at the end of 'n' compounding periods, assuming it will earn interest rate at the rate 'r' in each period.

PRESENT VALUE OF SINGLE SUM

Now let's just reverse the thought process. When we knew the present value (PV), interest rate (r) and the investment horizon (n), we could calculate the future value (FV). So if we know the future value, the applicable interest rate and the investment horizon, we should be able to derive the present value (PV) of the future investment.

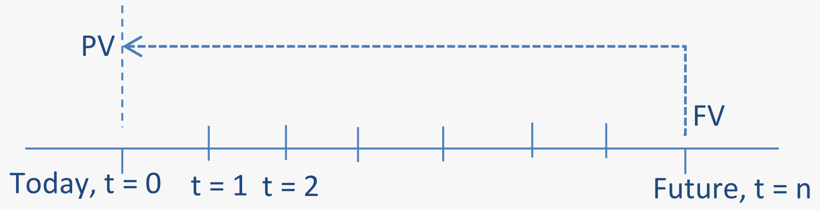
Present value of a single sum is today's value of cash that will be received in some future period. So this must equal the amount of money that must be invested today at given interest rate (rate of return) for a given period of time so that it grows to become equivalent to FV.

The process of computing PV of a cash flow is called *discounting* (recall we already also mentioned this earlier in the text). We discount the future cash flows back to present. The interest rate used to discount the cash flows is called *discount rate*. Please note that some people use the term *opportunity cost of capital*, *cost of capital*, hurdle rate, *weighted average cost of capital (WACC)* etc. to refer to discount rate.

For all practical purpose we will treat them all as one and the same. Since we already know the formula for FV, we just use high school algebra of cross multiplication and we have the formula for present value or PV.

$$PV = FV \div (1 + r)^n$$

$$PV = FV / (1 + r)^n$$



In the above formula the term 1/ (1 + r) ^{n} is also called present value factor or simply the discount factor.

The above formula is the single most important formula in the world of investment analysis. If you master this concept of present value, you have won half the battle of Petroleum Economics evaluation! The trillions dollar of investment are driven by this one formula! So please take time to understand this one single concept of time value of money and the remaining stuff will be just details.

ADDITIVITY OF PV & FV

Using the formula above for PV and FV we can easily determine the PV or FV of a given single amount. What we are presented with multiple amounts of cash flows occurring at different point in time. Can we use the formula above for finding PV and FV? Yes we can. We simply need to compute the PV of individual components and then add them together. Same stands for the FV. This is called additive of PV or FV.

Let's compute the PV for the series of cash flow as shown in table below.

Period	1	2	3
Cash flow ($)	1,000	3,000	5,000

Assume the cash flow to occur at the end of the period. Use discount rate r = 10% per period. To compute the FV of these cash flows at the end of period 3 all we have to do is sum the FV of individual cash flows of each period.

$$FV = FV_1 + FV_2 + FV_3$$

$$FV = 1000 \times (1+10\%)^2 + 3000 \times (1+10\%)^1 + 5000$$

$$FV = 9{,}510$$

Note that we compounded the cash flow of period 1 by only 2 and not 3.This is because the cash flow of period 1 occurred at the end of period 1 *or beginning of period 2*. So it has got only 2 periods left to grow or get compounded.

To compute the PV of these same cash flows we just sum the PV of individual cash flows of each period.

$$PV = PV_1 + PV_2 + PV_3$$

$$= 1000/(1+10\%)^1 + 3000/(1+10\%)^2 + 5000/(1+10\%)^3$$

$$= 1000/1.1 + 3000/1.21 + 5000/1.33$$

$$PV = 7{,}145$$

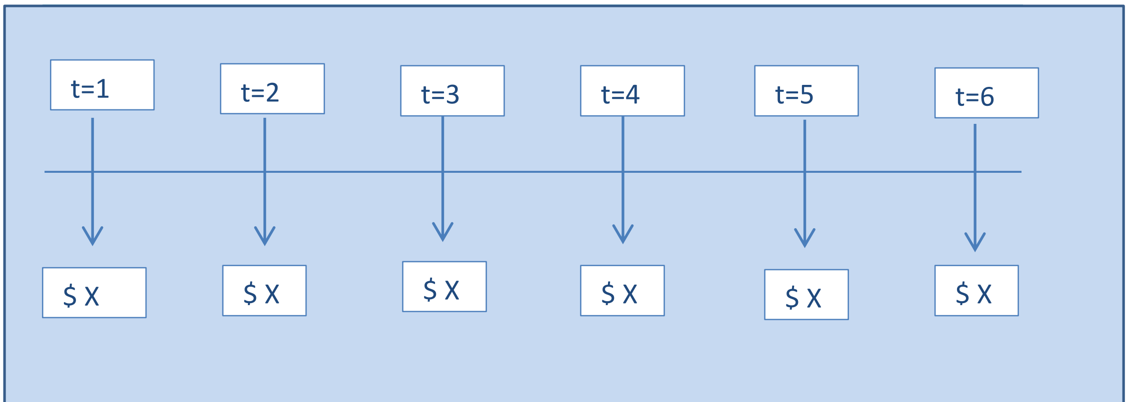
For the further reading ahead let’s use the notation NCF for the net cash flow (sum of all the cash flow over all time period). We will use CF to denote periodic cash flow.

ANNUITIES

An annuity is a stream of equal cash flows occurring at equal periodicity. For example if you are depositing $1,000 every month in your savings a/c and plan to do so for next 10 or 20 or so years, this stream of cash flow is an annuity. Calculating the PV or FV of an annuity is exactly similar to calculating the PV or FV of single sum. Just that in the case of an annuity we have to compute the PV or FV of not just one single sum, but many such sums. So mathematically, it’s just additions of the PV or FV of all such single sums.

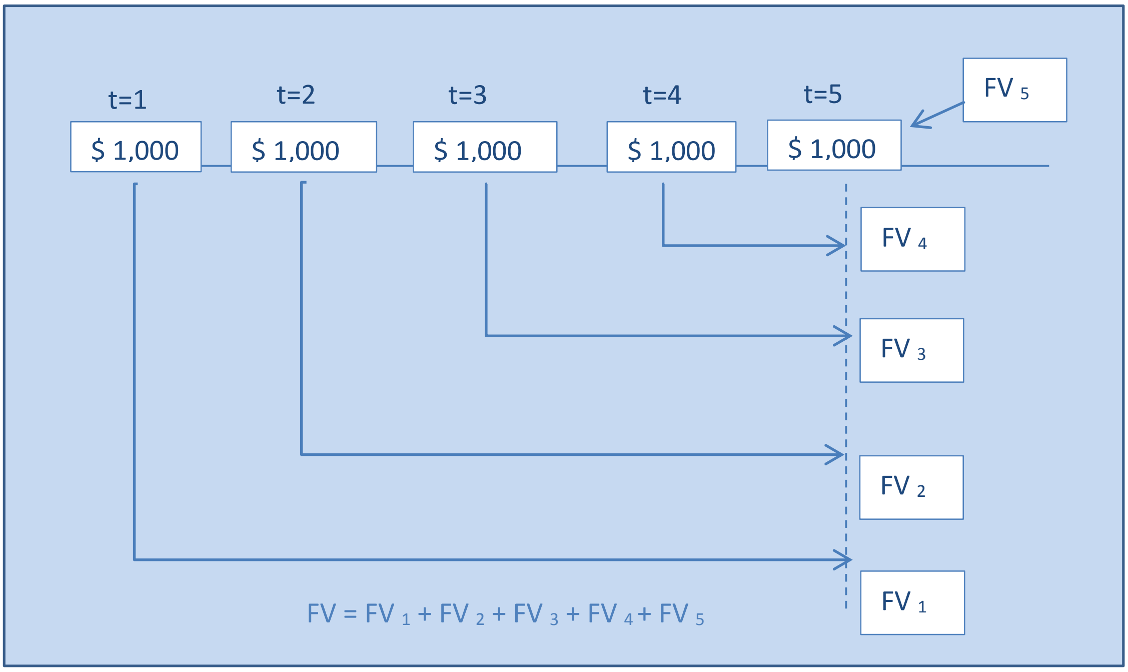
FV OF ANNUITIES

Assume an investment made by you generates a cash flow for five years as presented in the table below



Cash flow example in an annuity

Assume these cash flows are paid at the end of each period. Use discount rate r = 10% per period. To compute the FV of these cash flows at the end of period 5 all we have to do is sum the FV of individual cash flows of each period.



$FV = FV_1 + FV_2 + FV_3 + FV_4 + FV_5$

$FV = 1000 \times (1+10\%)^4 + 1000 \times (1+10\%)^3 + 1000 \times (1+10\%)^2 + 1000 \times (1+10\%)^1 + 1000$

$FV = 6{,}105$

Now this is was no different than what we did in the case of uneven cash flow series in the previous example. *The only differentiating factor is that each periods CF are same.*

And we can use this to simplify the calculation using shortcut formula for annuities. Notice that sum of individual FV is actually a Geometric Progression. And we can thus use the formula for summing a geometric progression.

It can be shown that FV of annuity equals:

FV of an annuity:

$$FV = CF \times [\frac{(1+r)^n - 1}{r}]$$

CF = fixed payment or receipts of each period

r = assumed interest rate

n +1 = total number of annuity payment

PV OF ANNUITIES

To compute the PV of above annuity at the beginning of period 1 (or end of period 0) we simply do the summation of PV of individual cash flows of each period.

$PV = PV_1 + PV_2 + PV_3 + PV_4 + PV_5$

$PV = 1000 / (1+10\%)^1 + 1000 / (1+10\%)^2 + 1000 / (1+10\%)^3 + 1000 / (1+10\%)^4 + 1000/ (1+10\%)^5$

PV = 3,791

Now doing this also is no different than what we did in the case of uneven cash flow series in the previous example. *The only differentiating factor is that each periods CF are same.* Just as in the case of finding FV we can use summation formula of a geometric series to compute a PV of an annuity.

It can be shown that PV of annuity equals:

PV of an annuity:

$$PV = CF \times [\frac{1-(1+r)^{-n}}{r}]$$

CF = fixed payment or receipts of each period

r = assumed interest rate

n +1 = total number of annuity payment

Now let's move to a special case of annuities, perpetuity. But before that let me ask you a simple question. Assume you are receiving $1,000 at the end of each year. This income stream of yours is never going to stop. You will be earning $1,000 year after year ad infinitum. What is the value to you *today*, of all these infinite series of $1,000 that you will receive in the future? This is what we are going to answer next.

PV OF PERPETUITY

Perpetuity is a type of cash flow of fixed amount that occurs at regular periodic interval just like annuities, but for infinite time. Again to find the PV is actually a simple task of finding the PV of each individual stream of cash flow, but for infinite such cash flows.

PV of perpetuity: $PV = CF / r$

CF = the fixed payment each period
r = interest rate or discount factor

The above formula is simple to derive. Since perpetuity is a special case of annuity where n =∞ (infinity), just plug in the value of n as ∞ in the formula of annuity and due to mathematics of limit theory the formula becomes as shown above.

Based on the formula above, the answer to my question in previous paragraph then is simple and here is how to work it out. You are receiving a perpetuity whose value in today's terms = $1,000/ (10%) = $10,000. Now that may sound bizarre to you but the sum of an infinite series is a finite number and such is the wonder of mathematics!

TEST YOU TIME VALUE CONCEPTS

I hope you have gained some perspective on the PV and FV of a cash flow stream. This is the most critical concept to understand and that's why we are focusing so much time on this topic. We really want you to master it not just mechanically but visually as well. Basically any problem associated with the time value of money can be easily solved if you can visualize yourself time travelling! Moving back and forth in the time. The following exercise should help you clear that point.

Assume you want to do a PhD (if you already are, then bravo! Let's assume you want to do another PhD!) in 5 years' time from now. You calculate that you will need $1,000 each month for your course and living expense. You expect to earn an interest rate of 10% annual on all of your savings in.

So here is the problem. How much should you invest in your savings each year for the next 5 years (at the beginning of each year), such that when you start your PhD program, the investments should be able to cover your annual expenses of $12,000 for the entire duration of the course, which will be 4 years.

Try to figure out a way and solve this investment problem. We don't want think first before you check out our answer or methodology. So don't move to next topic until you have thought enough about this problem. The solution is toward the end of next topic. So take out pen and paper, draw a time line, chalk out the cash inflows and outflows timings on the time line and try to resolves the situation!

NPV – THE GOLD STANDARD

We hope you had good time thinking about the problem we presented above! Here we discuss one final concept of this section, before showing you one possible way to answer our question.

NPV stands for Net Present Value of an investment or series of cash flows.

The net present value (from here onward we will call it NPV) of an investment is the present value of all expected cash inflows *minus* the present value of expected cash outflows. To find the NPV the project cash flows must be discounted at the *appropriate cost of capital.*

NPV represent value of the investment on the reference date (generally 'today' or at the present). This is the most widely used measure for investment decision worldwide by CEO's, CFO's and other key decision making authorities. So learn it well!

Steps to calculate the NPV:

1. Measure all costs (cash-outflows) of the project
2. Measure all benefits (cash-inflows) of the project
3. Decide on the suitable discount rate or factor (ask the corporate finance team)
4. Find the PV of all cash flows. Cash-inflows are positive and will increase the NPV; cash-outflows are negative and thus reduce the NPV.

NPV Formula:

$$NPV = \sum_{t=0}^{t=n} \frac{CF_t}{(1+r)^t}$$

CF_t = net cash flow in the time period t

t = time period

n = total number of periods

r = discount rate or cost of capital or opportunity cost of capital or hurdle rate

Take an example. A project needs $30 million as upfront investment. Due to this investment the project generates $10, $25, $20, and $15 million as positive cash flows at the end of each year. What's the value of this project on the inception date at an assumed discount rate of 12%?

In this case NPV is equal to PV of all cash flows minus the initial cash outflow at time t=0. We just need to find the PV of all cash flows and sum it up which gives us the project NPV.

NPV = -30 + 10/ (1.12) + 25/ $(1.12)^2$ + 20/ $(1.12)^3$ + 15/ $(1.12)^4$
NPV = $35 million

SOLUTION OF THE INVESTMENT PROBLEM

OK! So here is the solution of the problem we talked about previously. Let's summaries the problem:

How much should you invest in your savings each year for the next 5 years (at the beginning of each year), such that when you start your PhD program (beginning of 6th year), the investments should be able to cover your annual expenses of $12,000 for the entire duration of the course, which will be 4 years. Assume you expenses are at the end of each year.

First draw a time line and show your cash flow requirement. It should be somewhat as shown below. All X are investments required in the first 5 years (at the beginning of the each year). All Y are the cash that you must have in each year (at the end of the year) for your spending needs.

Year 1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9
-X	-X	-X	-X	-X				
					Y	Y	Y	Y

The problem has two main parts. First you need to calculate the PV of your entire future cash requirement. That's easy to find. We have a cash outflow of Y = $12,000 each year for 4 years. The PV of that equals $38,038 at the beginning of the years from where the expenses start (i.e. at the beginning of year 6 or end of year 5). Thus the FV of all the individual investments (=X) must equal the PV of $38,038. Now using Goal Seek function of Excel we can find the value of X such that FV of all X equals $38,038. The answer using Excel Goal Seek comes around to $5,664.